Passive Controller Design for Swing Phase of a Single Axis Above-Knee Prosthesis

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Abstract

In this research we design a passive controller for an above knee prosthesis. The controller is a linear spring and damper for swing phase motion, parameters of which determined via optimization of adjustment of the prosthesis shank motion with a desired shank angle trajectory comes from experimental data. In this way, we exerted a certain thigh motion, hip movement included, into the system as its input and found a set of unknown parameters such that the model output gets as close as possible to a certain output which is shank motion in normal level walking. Also a robustness analysis of the controller with respect to input motion changes included to show the effectiveness in real application.

Keywords : swing phase, above knee prosthesis, passive controller, optimization.

Introduction

Since the parameters of prosthetic knee have an essential effect on its function, they should be studied to make a prosthetic gate as close as possible to an intact leg. Some studies have engaged in mathematical simulation of this parameters. [1] describes a control mechanism for an above knee leg through a mathematical modeling. An optimization of four bar knee mechanism is presented in [2]. Radcliffe [29] presented a mathematical model of a four bar polycentric knee. [18] appraised the influence of prosthetic knee inertial characteristics through a mathematical modeling. [5] investigated the optimal control of a two degree of freedom prosthetic knee. [2] carried out an optimization of the four-bar knee mechanisms. [13] discusses the problems associated with the use of inverse dynamics in prosthetic applications using a polycentric Knee. [14] uses a mathematical

modeling to investigate the hydraulic knee controller deterioration on gait.

In This study, we consider a simple mechanism of single axis knee joint, and our objective is to design an optimal passive controller for that on a basis of linear spring and viscous damper, such that achieve to an optimal track of the sound gait trajectory by the artificial leg mechanism.

Using MATLAB programming language, a computer simulation of a typical single axis prosthetic knee is carried out. Defining a proper cost function corresponding to the fitness of simulated prosthesis's output with the desired shank trajectory, then optimizing this cost function, one can find out a best set of controller parameters. Comparison of the desired shank motion with that of the mechanism loaded with designed controller parameters, shows the effectiveness of our design.

Problem Definition

In this paper we will design a simple controller as the knee torque producer, for an artificial leg. The artificial leg we are working on here is a one degree of freedom mechanism to be attached to the thigh of an amputee, the only joint of which is a single-axis revolute joint corresponding to the knee. The controller acts on knee and is passive, without any actuator and external energy supply, and will be designed on the basis of a spring/damper mechanism.

The objective of controller design is to produce a proper shank motion when thigh moves in a certain manner, during swing phase in the walking cycle. The thigh motion we will feed to the dynamic system of thigh-shank-controller is a definite trajectory comes from experimental data of normal straight walking.

Swing phase is the part of walking cycle during which there is no contact between foot and ground, thus, the thigh-shank system works as a complex pendulum. The upper link, thigh, has its motion, which is known, and the knee joint is supported with a controller the characteristics of which to be designed.

Prosthesis Model

In this work, we consider a planar single axis above-knee prosthesis model as shown in figure 1. H and K are the representatives of hip and knee joints in link-segment model of human body, respectively. The controller controls the knee flexion in the prosthesis, simulates the

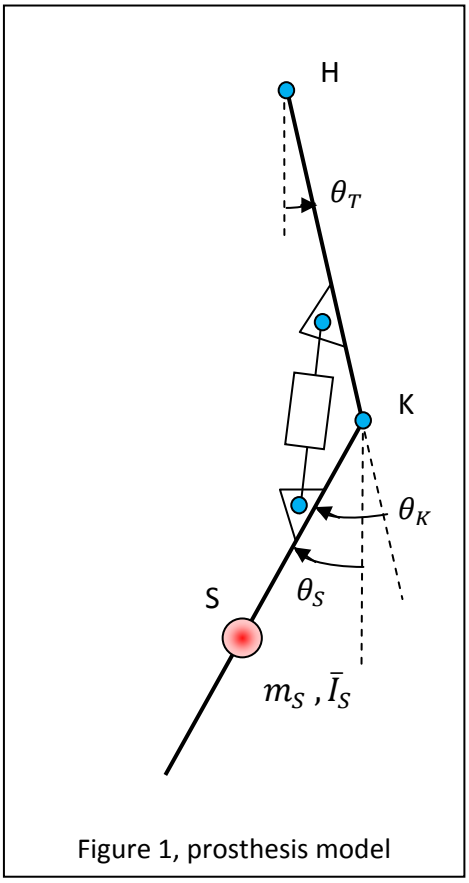

Figure 1, prosthesis model

role of knee muscles. Thigh is the link connecting H and K, and since its motion is known as an input when solving the equations of motion, and its dynamics is not desired, we don't care about its inertial properties. For shank, we assume its mass (m_S) concentrated at its center of gravity (*S*), and a moment of inertia ($\bar{I}_S$) with respect to the same point. The controller is a variable-length link upper joint and lower joint of which are connected to thigh and shank with a negligible offset from Hip-Knee line and Knee-S line, respectively.

The general force thigh exerts on shank through knee is called $\vec{f}_k$. According to our model, in which the controller is connected to thigh and shank both through pin joints, controller is a 2-force body, and its force $\vec{f}_c$ lies in the same direction with the controller.

Next, we devise the controller in our model to be a linear one, in which there is a linear relationship between controller force and controller length and its derivatives. More particularly we are looking for a linear spring and damper mechanism to control shank motion.

The coordinate system we use in the mathematical formulations also is determined in figure 1.

Mathematical Description

Equation of motion

Here we will drive the differential equation of motion for the described prosthesis shank. Writing Newton's second law for shank, we have $m_s\vec{a}_s = \sum \vec{F}_s = \vec{f}_k + \vec{f}_c + m_s\vec{g}$

$$\Rightarrow \vec{f}_k = m_s\vec{a}_s - \vec{f}_c - m_s\vec{g}$$

And Newton's second law for angular momentum :

$$-\bar{I}_s\ddot{\theta}_s = \sum \tau_s = -\vec{r}_{KS} \times \vec{f}_k - \vec{r}_{CS} \times \vec{f}_c$$
$$= -\vec{r}_{KS} \times (m_s\vec{a}_s - \vec{f}_c - m_s\vec{g}) - \vec{r}_{CS} \times \vec{f}_c$$
$$= m_s\vec{r}_{KS} \times (\vec{g} - \vec{a}_s) + \vec{r}_{KC} \times \vec{f}_c$$

Eq. 1

Starting with kinematic relations, one can easily obtain the linear acceleration of center of gravity of shank ($\vec{a}_s$) with respect to acceleration of hip ($\vec{a}_H$), thigh angle (θ_T) and its derivatives ($\dot{\theta_T}$, $\ddot{\theta}_T$) as follows:

$$\vec{a}_s = \vec{a}_H + r_{HK}\ddot{\theta}_T(cos\theta_T\hat{x} + sin\theta_T\hat{y}) - r_{HK}\dot{\theta_T}^2(sin\theta_T\hat{x} - cos\theta_T\hat{y}) + r_{KS}\ddot{\theta}_S(-cos\theta_S\hat{x} + sin\theta_S\hat{y}) + r_{KS}\dot{\theta_S}^2(sin\theta_S\hat{x} + cos\theta_S\hat{y})$$

And substituting $\vec{a}_s$ into Eq.1, we will have:

$$-\left(\frac{\bar{I}_s + m_s r_{KS}^2}{m_s r_{KS}}\right)\ddot{\theta}_s = (g + \ddot{y}_H + \dot{\theta}_T^2 r_{HK}\cos\theta_T + \ddot{\theta}_T r_{HK}\sin\theta_T)\sin\theta_s + \left(-\ddot{x}_H + \dot{\theta}_T^2 r_{HK}\sin\theta_T + \ddot{\theta}_T r_{HK}\cos\theta_T\right)\cos\theta_s + \frac{1}{m_s r_{KS}} r_{KC}\hat{s} \times \vec{f}_c$$

Eq. 2

Where $\hat{s} = -(sin\theta_S\hat{x} + cos\theta_S\hat{y})$

According to figure 2, we can write

$\vec{f}_c = f_c\, sin\beta\hat{x} - f_c cos\beta\hat{y}$

then

$\hat{s} \times \vec{f}_c = f_c(cos\beta\sin\theta_s + sin\beta\cos\theta_s) = f_c\sin(\theta_s + \beta)$

And substituting $\hat{s} \times \vec{f}_c$ in Eq.2 we will derive Eq.3 as follows:

$$-\left(\frac{\bar{I}_s + m_s r_{KS}^2}{m_s r_{KS}}\right)\ddot{\theta}_s = (g + \ddot{y}_H + \dot{\theta}_T^2 r_{HK}\cos\theta_T + \ddot{\theta}_T r_{HK}\sin\theta_T)\sin\theta_s + \left(-\ddot{x}_H + \dot{\theta}_T^2 r_{HK}\sin\theta_T + \ddot{\theta}_T r_{HK}\cos\theta_T\right)\cos\theta_s + \frac{1}{m_s r_{KS}} r_{KC}\sin(\theta_s + \beta) f_c$$

Eq.3

Defining

$$M = -\left(\frac{\bar{I}_s + m_s r_{KS}^2}{m_s r_{KS}}\right)$$
$$A = g + \ddot{y}_H + \dot{\theta}_T^2 r_{HK}\cos\theta_T + \ddot{\theta}_T r_{HK}\sin\theta_T$$
$$B = -\ddot{x}_H + \dot{\theta}_T^2 r_{HK}\sin\theta_T + \ddot{\theta}_T r_{HK}\cos\theta_T$$
$$D = \frac{r_{KC}}{m_s r_{KS}}$$

the Eq.3 may be rewritten in form of

$$M\ddot{\theta}_s = A(t)\sin\theta_s + B(t)\cos\theta_s + D\sin(\theta_s + \beta) f_c$$

Eq.4

A and B are functions of thigh motion, which is assumed to be known with respect to time, thus A and B are functions of time.

β in Eq.4 is a geometric function of knee angle θ_K, where $\theta_K = \theta_T + \theta_S$ as follows:

$$\beta = \theta_T - \alpha$$
$$b^2 = a^2 + l_C^2 - 2al_C cos\alpha \Rightarrow cos\alpha = \frac{l_C^2 + a^2 - b^2}{2al_C}$$
$$l_C^2 = a^2 + b^2 + 2abcos\theta_K$$
$$\beta = \theta_T - cos^{-1}\left(\frac{2a^2 + 2abcos\theta_K}{2a\sqrt{a^2 + b^2 + 2abcos\theta_K}}\right)$$
$$\beta = \theta_T - cos^{-1}\left(\frac{1 + (b/a)cos\theta_K}{\sqrt{1 + (b/a)^2 + 2(b/a)cos\theta_K}}\right)$$

Where $b = r_{KC}$

Controller model

In this work we will design a controller to produce f_c during the swing phase of walking cycle. Our objective controller is a linear spring/damper based one as follows that can be described as:

$$f_c = -p(l_C - l_0) - d\dot{l_C}$$

Eq.5

Where l_C is the controller length and is a geometric function of knee angle θ_K as shown above. p is the spring coefficient , d is the viscous damper coefficient, and l_0 is the free length of the controller (the controller length corresponding to the free length of spring). These last three parameters of the controller are unknown and subject to design in this paper.

Optimization Problem

Let's define $x = [p\ , d\ , l_0]^T$ the unknown vector of the controller properties.

θ_S at the beginning of swing phase is known experimentally, thus for a certain x Eq.4 has a unique solution of Shank Angle trajectory $\theta_{sc}(t), t \in [t_0, t_f]$ depending on x which we show in our notations as $\widetilde{\theta_{sc}}(x)$. On the other hand we have a desired Shank Angle trajectory for swing phase, $\widetilde{\theta_{sd}}$ which comes from experimental data. The objective is to design a controller such that the shank motion in artificial prosthesis system gets as close as possible to this desired shank motion. This means an optimization problem must be solved here.

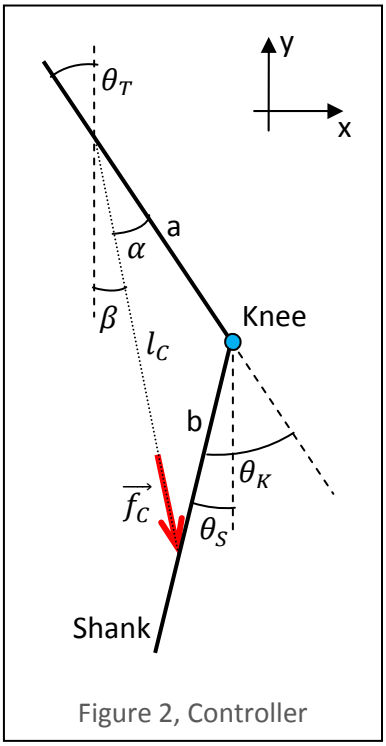

Figure 2, Controller

Let's define the cost function as

$$J(x) = \int_{t_0}^{t_f} \left(\theta_{sc}(x,t) - \theta_{sd}(t)\right)^2 . W(t) . dt$$

Eq.6

where t_0 and t_f are start time and end time of the swing phase, and W is a weight function. Then the optimization problem may be defined as

$$\min_x J(x)$$

Solution Method

J is a nonlinear but continuous scalar function of vector x, so must be solved numerically. Solving $\min_x J(x)$ is a typical optimization problem and can be solved using one of numerical algorithms. Physical constraint of the shank angle trajectory is, first, the shank angle at the beginning of swing phase is known from experimental data and this condition must hold in the solution, and second, there is a limitation on final shank angle at the end of swing phase. The first constrain is always satisfied in our solution method, since when solving the equation of motion for any x vector numerically, the initial shank angle is set as the desired amount. Second constraint on shank angle comes from the fact that at the end of swing phase, when heel approaches ground, knee angle must be almost zero, which means thigh and shank are in a straight line. It is common in knee prosthesis to prepare an extension stop bumper, active only at the end of swing cycle, e.g. the last 1 or 0.5 degrees of knee angle. This stop removes the requirement upon controller to have zero knee angle and zero knee angular velocity at the percent of 100 of the walking cycle, but extends the end-point condition to a wider region. Knee angle and its velocity must lie in a region around zero at the moment just before the end-point of walking cycle.

Some designers define a specific bumper and derive a moment-knee angle relationship for that and fed this additional moment into their equation of motion, then the end-point condition to be satisfied must be (zero, zero) for angle and velocity. But to give a greater weight to the cost optimization which is the desired and resultant angle trajectory adaptation, we prefer not to set a moment function for bumper, and after solving the optimization problem define a proper bumper.

Thus, in this work, we first numerically solve the optimization problem as an unconstrained one, then check the end-point situation. If knee angle and its velocity lies in an acceptable region such that a normal extension stop bumper may be used to satisfy the zero condition of percent 100 of walking cycle, that solution is OK. Otherwise, we will change the weight function in a manner that increases the cost of difference at last percents of walking cycle.

There are several numerical algorithms to solve such an unconstrained optimization problem, both in direct searching and gradient-based methods. What we use here is a subspace trust region method and is based on the interior-reflective Newton method described in [31],[32] (MUST INVOLVE), a simple yet powerful concept in optimization. Each iteration involves the approximate solution of a large linear system using the method of preconditioned conjugate gradients (PCG).

Numerical Solution

The experimental data for normal thigh and shank motion we use in this work is that of Zarrugh et. Al. 1976 [29]. Figure 3 shows x and y component of normalized hip position, thigh, knee, and shank angles for a normal walking at a cadence of 1.597 steps/sec, for percent of 50 to 100 of walking cycle, respectively.

Physical parameters of our model is as follows:

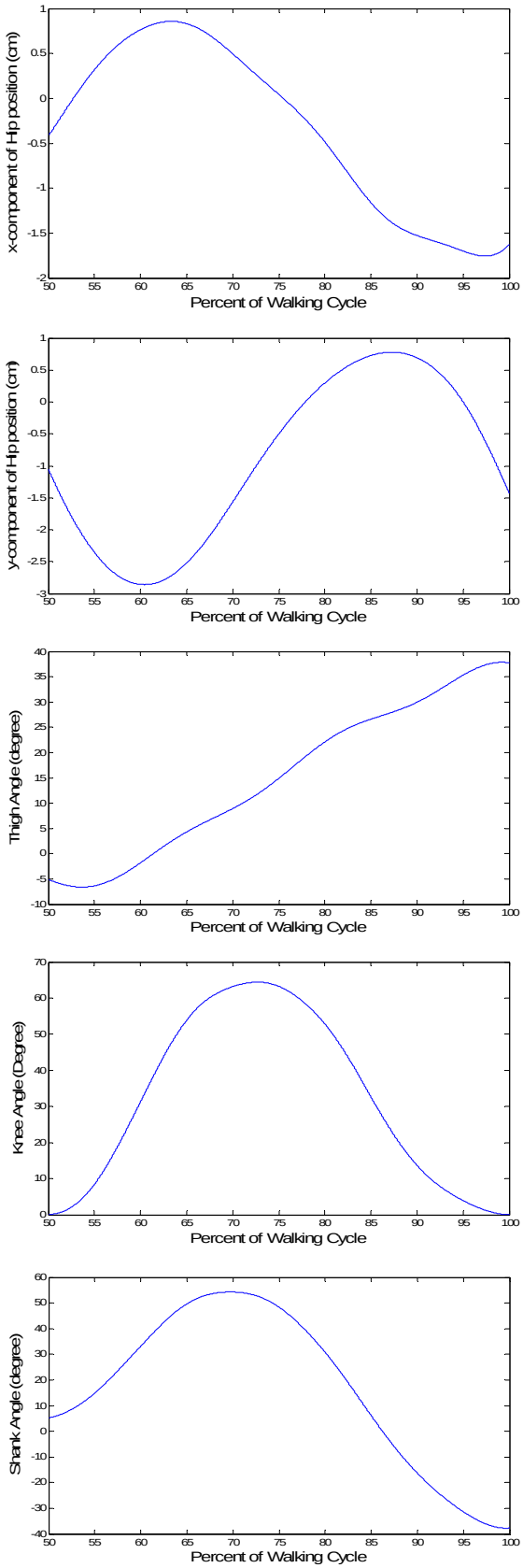

Figure 3 : hip position trajectory, thigh, knee and shank angle trajectory, as input signals

$$m_s = 2.36\ kg$$
$$\bar{I}_s = 0.136\ kg.m^2$$
$$r_{HK} = 43\ cm$$
$$r_{KS} = 18.6\ cm$$
$$a = 6\ cm$$
$$b = 8\ cm$$

M and D in Eq.2 are known from physical parameters of the model, $A(t)$ and $B(t)$ are know from hip and thigh motion, $f_c(x)$ is a function of l_C according to Eq.5, l_C and β both come from θ_k, or when θ_T is known, from θ_s. Thus, for a certain x solving Eq.4 for θ_s will give $\widetilde{\theta_{sc}}(x)$. Using the above experimental data of shank angle as the desired shank angle trajectory in Eq.6, cost function J will be found for that certain x. Starting from an initial guess for x, calculating gradient of J with respect to x and using Newton's interior reflective method the next guess will be selected. Optimization procedure continue until gradient of cost gets lower than an end-process threshold, which means a local optimum.

This optimization procedure is run throughout a MATLAB code. Starting from several initial guesses and comparing the results, finally we got the following optimal solution for the controller:

$x_{optimum} = (p \quad d \quad L_0)_{optimum} = (2019 \quad 2.19 \quad 0.38)$.

For this solution, the average difference between desired shank angle and calculated shank angle for the optimal x is about 13.3 degrees. This result is got for cost weight function of $W(t) = 1$. The desired and calculated knee motion are shown in figure 4.

For this found controller, knee angle at 98.12 percent of walking cycle will reach zero and its velocity will be 0.5 degrees/ms (for normal shank motion the velocity at the same point is about 0.1 degrees/ms), which is small enough to easily stop by a simple rubber bumper.

Right now we have optimized our controller parameters to have a proper shank motion when thigh moves in a certain manner. But an amputee does not always move his thigh according to the normal level walking, so it is

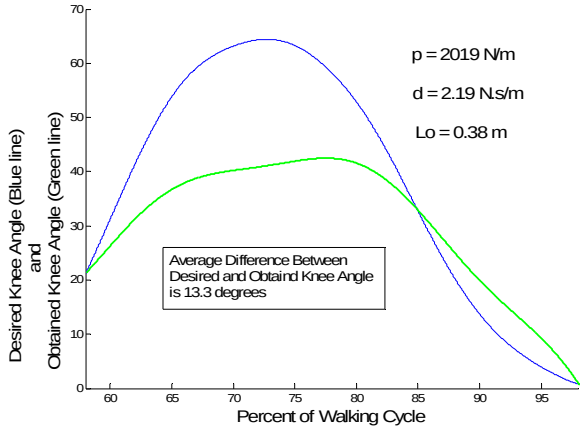

Figure 4 : desired (Blue) and resultant knee motion

necessary to check behavior of the prosthesis equipped with this controller, when thigh motion is not what we expect. This means that the system must be robust enough in some sense, with respect to deviations of thigh motion from its ideal trajectory.

For this purpose, we changed the exerted thigh angle trajectory using a deviator function. Solving the system with the designated controller for deviator functions of furrier type and different frequencies and amplitudes, but the maximum amplitude of 25 percent of maximum normal thigh angle, shows an acceptable robustness of this controller.

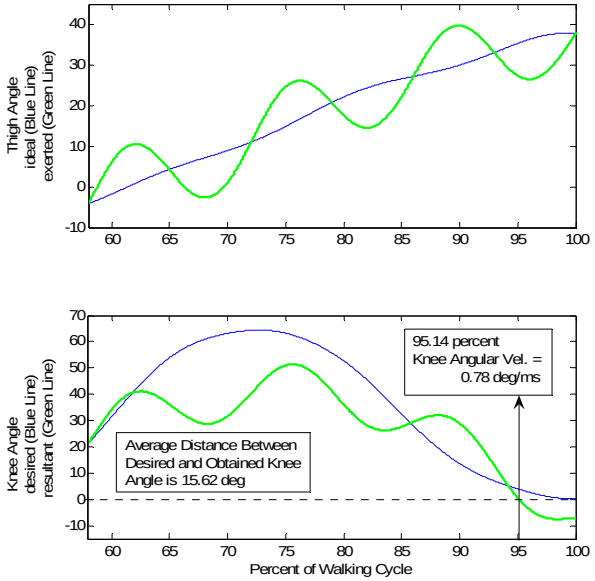

figure 5 : normal thigh and knee angle trajectory (blue), deviated thigh motion and its resultant knee motion when our controller is used.

Average difference between ideal shank angle and the system resultant one remains always less than 17 degrees, knee angle always remain in the range of [0,65] degrees, before end-cycle impulse, zero knee angle occurs in walking cycle percent of 93 to 99, and knee angular velocity at zero degree is always between less than 1.2degrees/ms, until 4th harmony. In higher frequencies, the impulse velocity increase, but those higher frequency components of thigh motion will not actually produce when a human walks.

For instance, for a thigh angle trajectory shown in figure 5-1, the resultant shank motion is shown in figure 5-2.

Discussion and Conclusion

In this research we designed a controller for an above knee prosthesis. The controller is a linear spring and damper, 3 parameters of which determined via optimization of adjustment of the prosthesis shank motion with a desired shank angle trajectory comes from experimental data. In this way, we exerted a certain thigh motion, hip movement included, into the system as its input and found a set of unknown parameters such that the model output gets as close as possible to a certain output which is shank motion in normal level walking. Since physical constraints seems to be easily satisfied automatically, the optimization problem solved as an unconstrained problem, and then the solution checked for constraints, which is mainly the endpoint condition of knee angle.

Also a robustness analysis included to consider if such a controller for such a prosthesis system is practical and applicable in real or not. An amputee wearing this prosthesis does not necessarily exert always the normal thigh motion as we expect. So the controller must be robust enough to deviations of thigh motion, and stable and limited under any limited actual thigh motion. This concept of stability is always satisfied since the controller is passive and actuator free, and the robustness with respect to thigh motion deviations checked for the designated controller.

In this research we dealt with a simple prosthesis model, and a simple-type linear controller. This work is done to draw a sketch of a concrete method for future researches on more complicated prosthesis systems taking advantage of more sophisticated and effective controllers with several unknown parameters.

This work should continue with considering performance of the prosthesis equipped with the designated controller in other usual tasks, like ramp walking, step climbing, jumping, etc, to show its abilities and drawbacks in daily life. Then, one should change the prosthesis model design and controller scheme to increase its performance.

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